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Letter to the Editor

Comments on "Dual analysis for path integrals and bounds for crack parameter"

C.C. Wu, Q.Z. Xiao, G. Yagawa

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In a recent paper, Wu et al. (1998) introduced a 'new' dual path independent integral I^* defined as follows:

$$I^* = \int_\Gamma \left[-B(\sigma_{ij})\,\mathrm{d}x_2 + u_i \frac{\partial \sigma_{ij}}{\partial x_1} n_j\,\mathrm{d}s + \frac{\partial (u_i \sigma_{i2})}{\partial x_j}\,\mathrm{d}x_j \right] \tag{1}$$

where Γ is the usual path around the crack tip, with the end points $A_\pm$ on the crack faces, lying on the negative Ox axis, $B(\sigma)$ is the complementary energy function. The stress free conditions on the crack faces read $\sigma_{12} = \sigma_{22} = 0$.

This I^*-integral is proved to be equal to Rice's J-integral

$$I^* = J = \int_\Gamma \left[W(\varepsilon_{ij})\,\mathrm{d}x_2 - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_1}\,\mathrm{d}s \right] \tag{2}$$

and is equal to the complementary energy release rate (a: crack length):

$$I^* = \frac{\mathrm{d}\pi_c}{\mathrm{d}a}. \tag{3}$$

The properties (2) and (3) are exactly those of the dual I-integral of Bui (1974), defined as

$$I = \int_\Gamma \left[-B(\sigma_{ij})\,\mathrm{d}x_2 + u_i \frac{\partial \sigma_{ij}}{\partial x_1} n_j\,\mathrm{d}s \right]. \tag{4}$$

The authors *showed* that "... the value of I is not identical to that of J and the mechanical sense of I is different from that of J as well". We conclude that somewhere there is a mistake, either in Bui's or Wu et al.'s results.

A careful check of the proof given in their paper shows that (2) and (3) are *correct*. The difference between I^* and I is given by the third additional term in (1)

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$$I^*-I=\int_\Gamma \frac{\partial(u_i\sigma_{i2})}{\partial x_j}\,\mathrm{d}x_j. \tag{5}$$

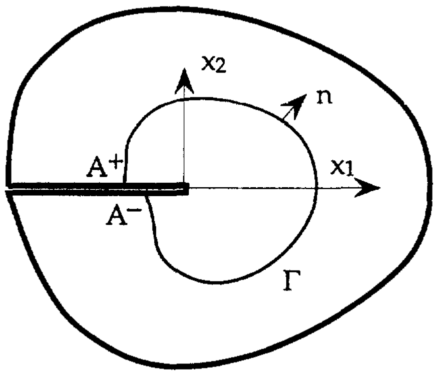
It is precisely here that we find a mistake in the *proof* by Wu et al. (1998). The integrant in (5) is an *exact differential*:

$$I^*-I=\int_\Gamma \mathrm{d}[u_i\sigma_{i2}]. \tag{6}$$

This difference is given by $I^*-I=[u_i\sigma_{i2}](a^+)-[u_i\sigma_{i2}])A^-)=0$. Therefore, there is *no difference* between the "new" dual I^*-integral and Bui's I-integral. Since $I^*=J$, the relation (6) can be found in classical text books on Fracture Mechanics. For example, one can find explicitly (6) in the recent book of Bazant and Planas (1998), in an exercise for students, page 94:

Exercise 4.20 Show that $I=J-\int_\Gamma \mathrm{d}[u_i\sigma_{i2}]$ and demonstrate that $I=J=G$.

In conclusion, there is nothing new in the I^*-integral proposed by Wu et al. (1998). The additional term is identical to zero. The I^*-integral is nothing but the dual I-integral of Bui (1974).



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